# Total Integrated Scatter (*TIS*)

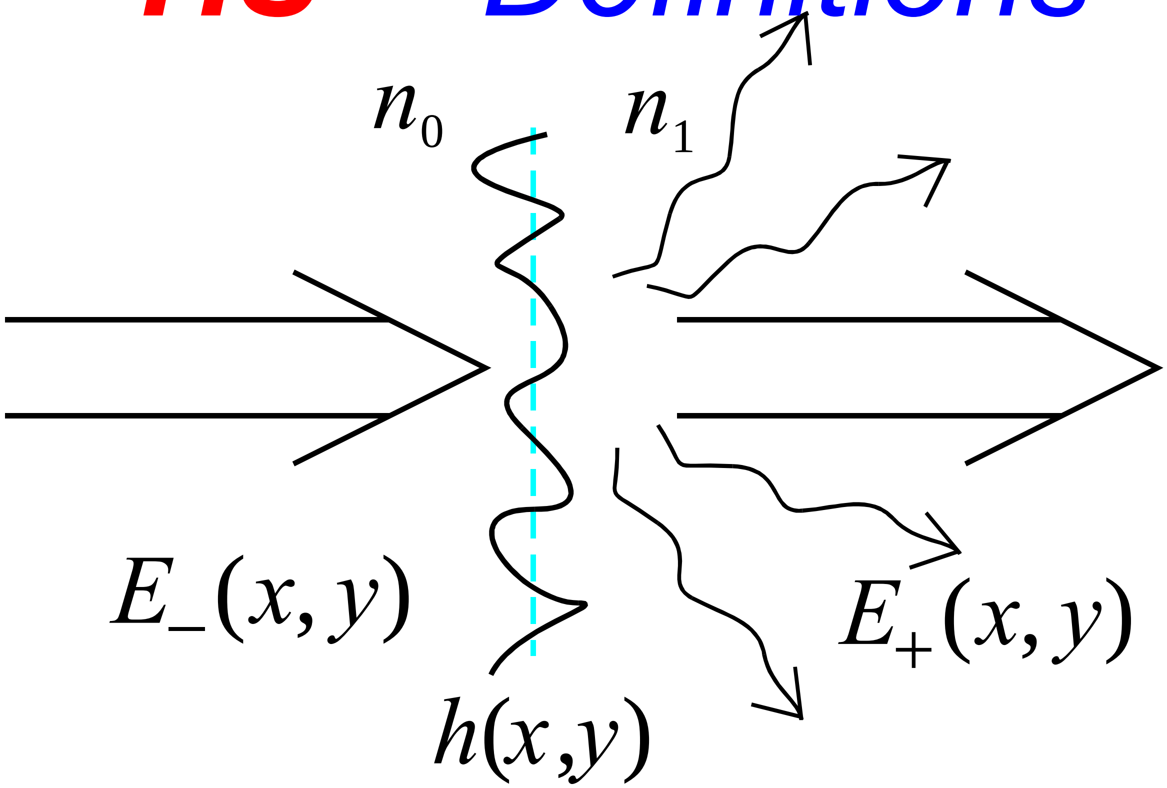
$$\boldsymbol{TIS} = \Delta n^2 \left( \frac{2\pi}{\lambda} \right)^2 \overline{\Delta h^2}$$

Given a refractive surface and reflective surface each with RMS height fluctuations of $\Delta h$, the ***TIS***, for the refractive surface (i.e. lens) will be 16 times less (assuming $n$=1.5 or $\Delta n$=.5) than the reflective surface (i.e. mirror).

A refractive surface (i.e. lens) with RMS roughness of 20nm has the same ***TIS*** as a reflective surface (i.e. mirror) with an RMS roughness of 5nm.

# *Total Integrated Scatter*

## TIS -- Definitions



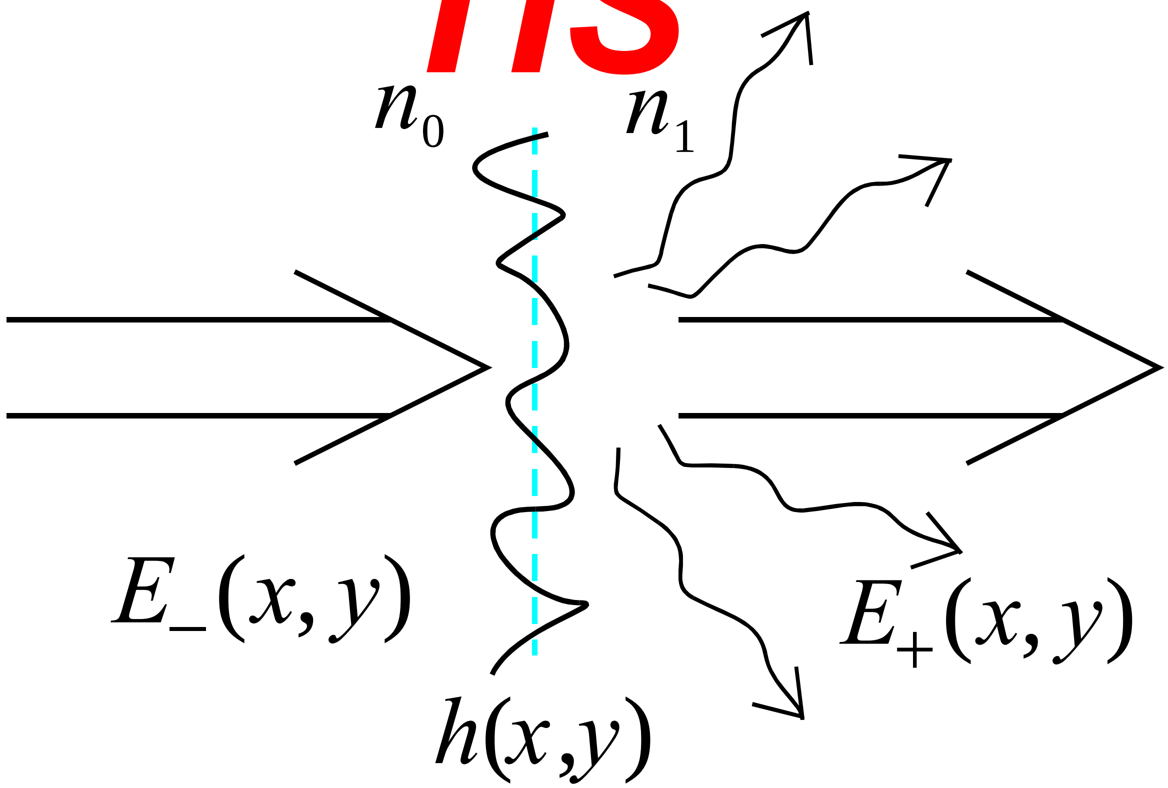

$E_{\_}(x,y)$ : field before interface

$E_{\_}(x,y)$ : field after interface

$h(x,y)$ : surface roughness, variation in surface hieght about, random spatial function, zero mean, variance $\overline{\Delta h^2}$

$t(x,y)$ : transmitance of surface, introduces random phase fluctuations due to $h(x,y)$

# TIS




$$E_+(x,y) = t(x,y) \cdot E_-(x,y)$$

where $$t(x,y) = t_0 e^{i\phi(x,y)}$$

$$\phi(x,y) = \Delta n \frac{2\pi}{\lambda} h(x,y)$$

Hence

$$t(x,y) \approx t_0 \left[ 1 + i\,\Delta n \frac{2\pi}{\lambda} h(x,y) - \tfrac{1}{2}\left( \Delta n \frac{2\pi}{\lambda} h(x,y) \right)^2 \right]$$

Note: $h(x,y)$ is the random surface variations about the mean.

# *Correlation Function*
and
# **Angular Power Spectrum**

$$C_{E_-}(\varepsilon_x,\varepsilon_y)=\overline{E_-^*(x,y)\cdot E_-(x+\varepsilon_x,y+\varepsilon_y)}$$

$$C_t(\varepsilon_x,\varepsilon_y)=\overline{t^*(x,y)\cdot t(x+\varepsilon_x,y+\varepsilon_y)}$$

$$=t_o{}^2\left[1+\Delta n^2\left(\frac{2\pi}{\lambda}\right)^2\overline{\Delta h^2}\left(\gamma_h(\varepsilon_x,\varepsilon_y)-1\right)\right]$$

where $\overline{\Delta h^2}$ is the variance about the mean

and $\gamma_h(\varepsilon_x,\varepsilon_y)$ is the normalizm correlation function of $h(x,y)$

$$C_{E_+}(\varepsilon_x,\varepsilon_y)=C_t(\varepsilon_x,\varepsilon_y)\cdot C_{E_-}(\varepsilon_x,\varepsilon_y)$$

# Correlation Function
and
# *Angular Power Spectrum*

$$L_{-}(f_x, f_y) = \mathcal{F}^{(2)}\left\{C_{E_{-}}(\varepsilon_x, \varepsilon_y)\right\}$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C_{E_{-}}(\varepsilon_x, \varepsilon_y)\exp\left[i2\pi\left(f_x\varepsilon_x + f_y\varepsilon_y\right)\right] d\varepsilon_x d\varepsilon_y$$

$$L_{+}(f_x, f_y) = L_{-}(f_x, f_y) ** T(f_x, f_y)$$

$$T(f_x, f_y) = \mathcal{F}^{(2)}\left\{C_t(\varepsilon_x, \varepsilon_y)\right\}$$

$$= t_o^{\,2}\left[\underbrace{\left(1 - \Delta n^2\left(\frac{2\pi}{\lambda}\right)^2 \overline{\Delta h^2}\right)\delta(f_x, f_y)}_{\textit{unscattered}} + \underbrace{\Delta n^2\left(\frac{2\pi}{\lambda}\right)^2 \overline{\Delta h^2}\,\Gamma_h(f_x, f_y)}_{\textit{scattered}}\right]$$

# Directional Flux

$$L(f_x, f_y)\, df_x\, df_y \quad : \begin{cases} \text{flux per unit area traveling within} \\ \text{cosine directions } f_x \;\&\; f_x + df_x \\ \text{and } f_y \;\&\; f_y + df_y \end{cases}$$

Note: $df_x df_y = \frac{1}{\lambda^2} \cos(\theta)\, d\Omega = \sqrt{\frac{1}{\lambda^2} - f_x^2 - f_y^2}\; d\Omega$

$d\Omega$: differential solid angle associated with $df_x df_y$

# Total Radiative Exitance (Flux per Unit Area)

$$M = \iint_{f_x^2 + f_y^2 \le \frac{1}{\lambda^2}} L(f_x, f_y)\, df_x\, df_y$$

# *Total Integrated Scatter*

# ***TIS***

$$TIS = \Delta n^2 \left(\frac{2\pi}{\lambda}\right)^2 \overline{\Delta h^2} \iint_{f_x^2+f_y^2\le\frac{1}{\lambda^2}} \Gamma(f_x, f_y)\, df_x\, df_y$$

In general $\displaystyle\iint_{f_x^2+f_y^2\le\frac{1}{\lambda^2}} \Gamma(f_x, f_y)\, df_x\, df_y \le 1$

For reflection: $\Delta n = 2$

For refraction: $\Delta n \approx .5$

Hence the total integrated scatter (***TIS***) is approximately sixteen times less for a refractive surface compared to a reflective surface with the same statistical surface roughness.

# *TIS*--Special Case

Separable Gaussian Random Surface

$$\gamma_h(\varepsilon_x, \varepsilon_y) = \exp\left(-\pi \frac{\varepsilon_x^2}{\sigma_x^2}\right) \exp\left(-\pi \frac{\varepsilon_y^2}{\sigma_y^2}\right)$$

Hence

$$\Gamma_h(f_x, f_y) = \sigma_x \sigma_y \exp(-\pi\, \sigma_x^2 f_x^2) \exp(-\pi\, \sigma_y^2 f_y^2)$$

Note the following limits

$$\iint_{f_x^2 + f_y^2 \le \frac{1}{\lambda^2}} \Gamma(f_x, f_y)\, df_x\, df_y \approx \begin{cases} 1 & \sigma_x >> \lambda \,\&\, \sigma_y >> \lambda \\ \dfrac{2\sigma_x}{\lambda} & \sigma_x << \lambda \,\&\, \sigma_y >> \lambda \\ \dfrac{\pi \sigma_x \sigma_y}{\lambda^2} & \sigma_x << \lambda \,\&\, \sigma_y << \lambda \end{cases}$$